

# Warm up

Expand and simplify the following expressions:

$$1 \quad 2(\mathbf{a} + \mathbf{b}) + \mathbf{b} = ?$$

$$2 \quad \frac{1}{2}(\mathbf{a} - \mathbf{b}) + \mathbf{b} = ?$$

$$3 \quad \mathbf{a} + \frac{1}{2}(\mathbf{a} + \mathbf{b}) = ?$$

$$4 \quad \mathbf{a} - 2(\mathbf{a} - 2\mathbf{b}) = ?$$

$$5 \quad \frac{1}{3}(\mathbf{a} + 2\mathbf{b}) - \mathbf{b} = ?$$

# Vectors Quiz!

Mini whiteboards  
ready?

# Vectors Quiz!

- a.  $\overrightarrow{RQ}$
- b.  $\overrightarrow{PR}$**
- c.  $\overrightarrow{QP}$
- d.  $\overrightarrow{RP}$

What is  $\overrightarrow{PQ} + \overrightarrow{QR}$ ?

2. This diagram shows vectors  $\mathbf{a}$  and  $\mathbf{b}$ .



- a.  $\mathbf{a} + \mathbf{b}$
- b.  $\mathbf{a} + 2\mathbf{b}$
- c.  $\mathbf{a} - \mathbf{b}$
- d.  $\mathbf{b} - \mathbf{a}$

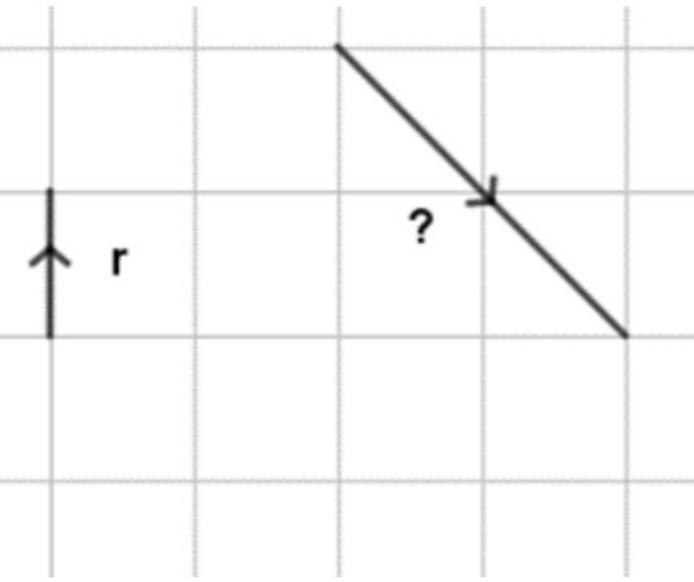
Which vector is shown in this diagram to the right?



# Vectors Quiz!

- a.  $p - q$
- b.  $2a$
- c.  $-a$
- d.  $q - p$

If  $\overrightarrow{PQ} = a$ , what is  $\overrightarrow{QP}$ ?



a.  $p - r$

b.  $p - 2r$

c.  $p + 2r$

d.  $p + r$

4. What is the missing vector?

# Vectors Quiz!

- a.  $\begin{pmatrix} 10 \\ 3 \end{pmatrix}$
- b.  $\begin{pmatrix} 24 \\ -18 \end{pmatrix}$
- c.  $\begin{pmatrix} 64 \\ -36 \end{pmatrix}$
- d.  $\begin{pmatrix} 10 \\ 9 \end{pmatrix}$

5. Complete  
this vector  
addition

$$\begin{pmatrix} 6 \\ -3 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix} =$$

# Vectors Quiz!

- 1) They are parallel
- 2) AB is k times the length of BC.

If  $\overrightarrow{AB} = k\overrightarrow{BC}$ , what do we know about lines AB and BC?

Write down two things.

# Vectors Quiz!

a.  $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$

b.  $\begin{pmatrix} 2 \\ -12 \end{pmatrix}$

c.  $\begin{pmatrix} -3 \\ 2 \\ 7 \\ 5 \end{pmatrix}$

d.  $\begin{pmatrix} 10 \\ -2 \end{pmatrix}$

7. Complete  
this vector  
subtraction

$$\begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} -6 \\ 7 \end{pmatrix} =$$

# Vectors Quiz!

a.  $\begin{pmatrix} 10 \\ -4 \end{pmatrix}$

b.  $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$

c.  $\begin{pmatrix} 4 \\ 13 \end{pmatrix}$

d.  $\begin{pmatrix} 25 \\ -22 \end{pmatrix}$

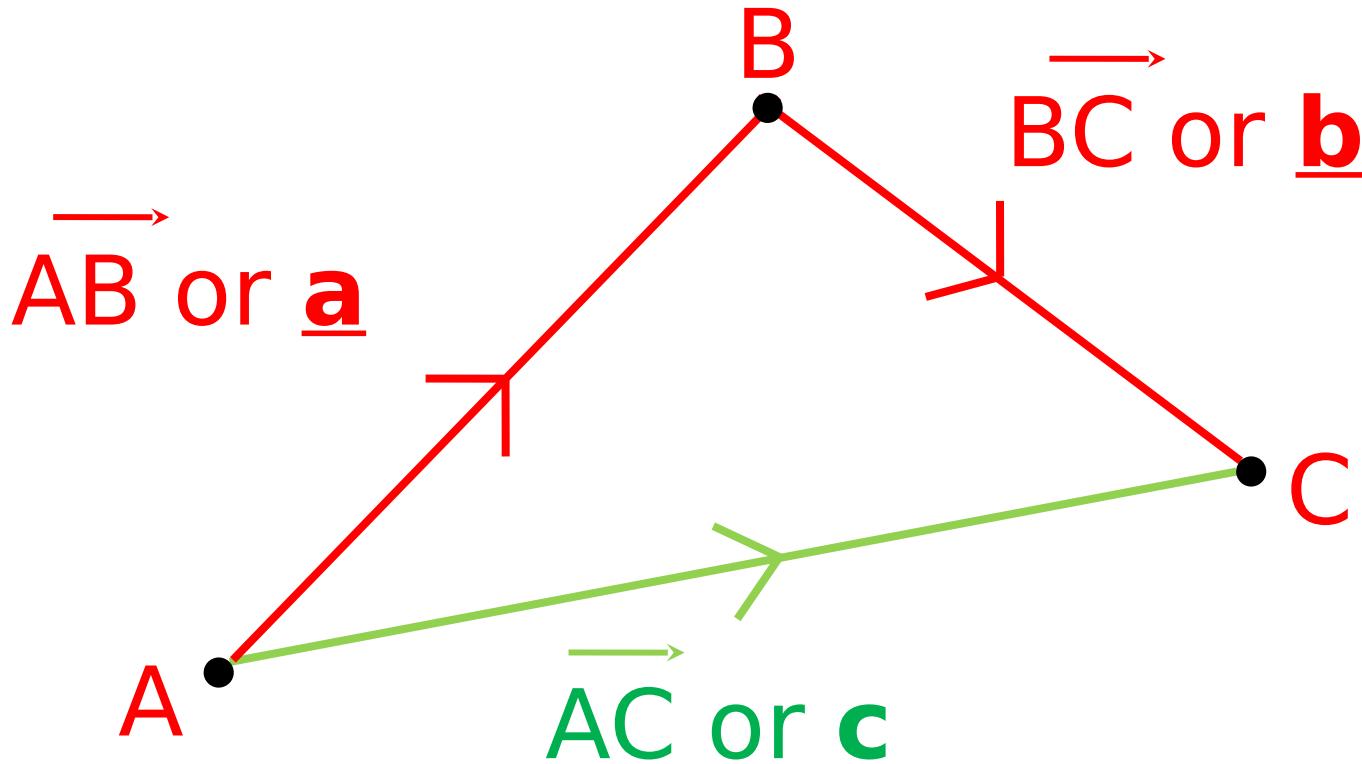
5.  $\mathbf{g} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}, \mathbf{h} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

Work out the vector  $2\mathbf{g} + \mathbf{h}$ .

None of  
them!

$$\begin{pmatrix} 4 \\ -13 \end{pmatrix}$$

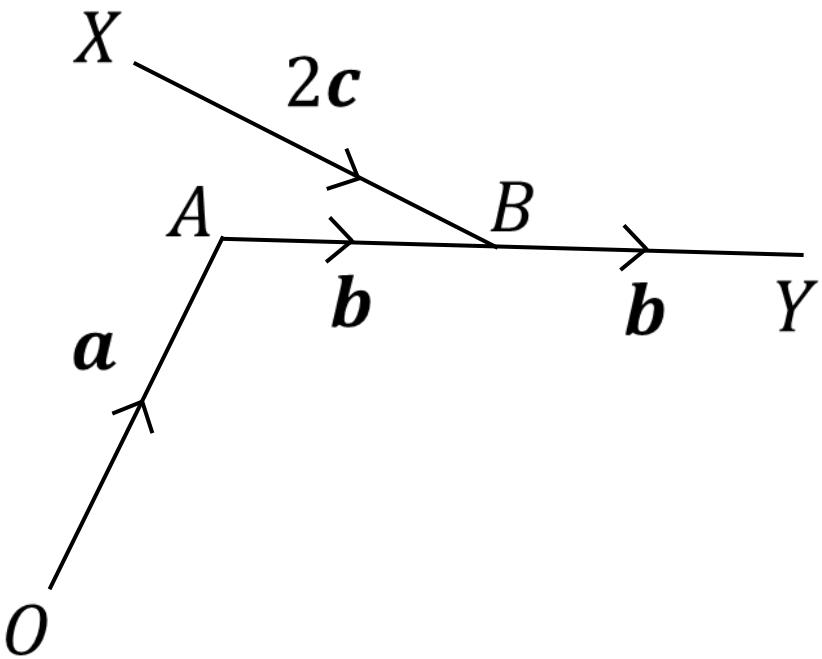
# Resultant Vectors



A resultant vector is a single vector which is equivalent to a set of vectors

$$\vec{AC} = \vec{AB} + \vec{BC} \quad \text{or} \quad \underline{c} = \underline{a} + \underline{b}$$

# Adding and Subtracting Vectors

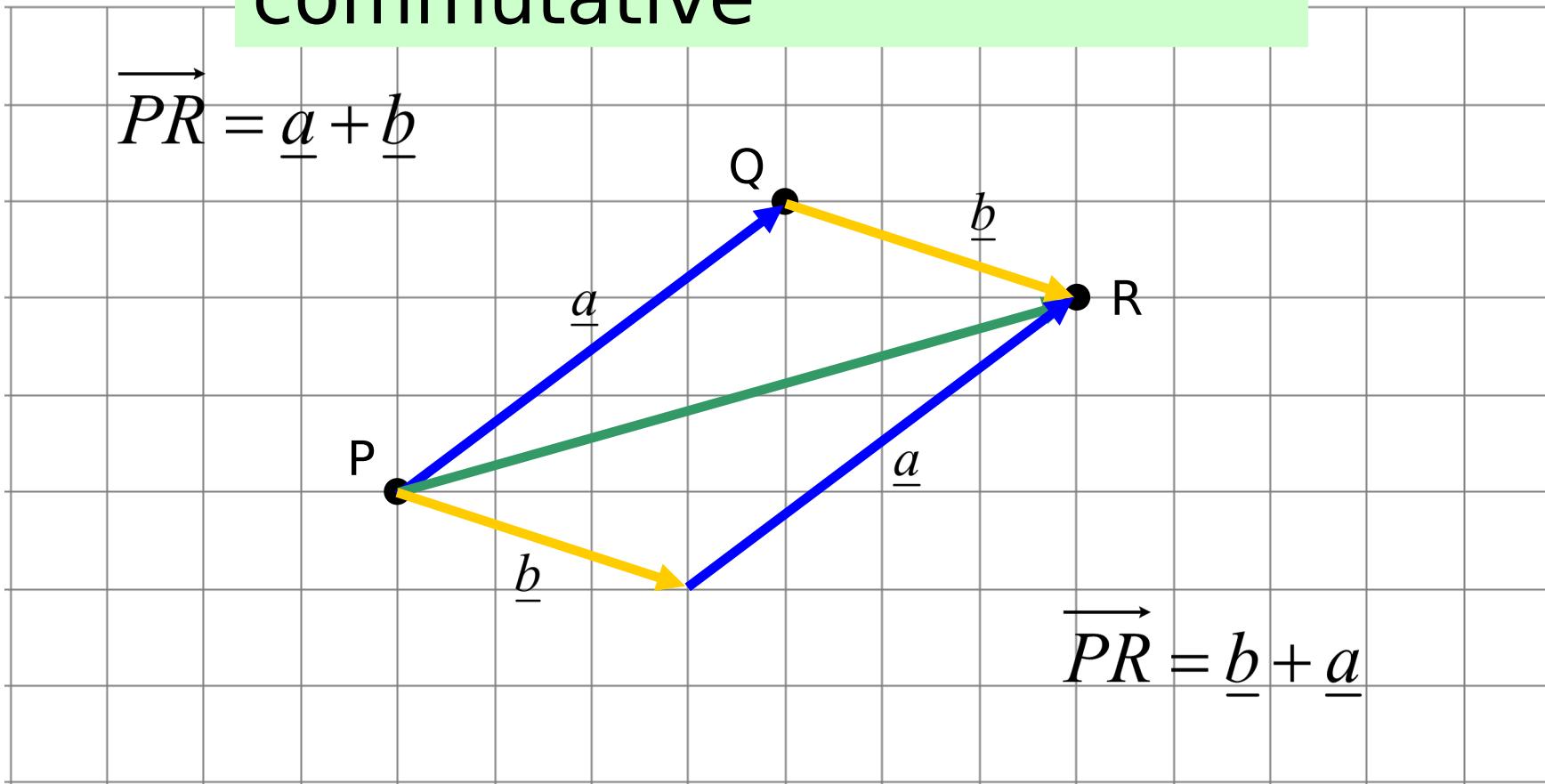


If  $\overrightarrow{OA} = \mathbf{a}$ ,  $\overrightarrow{AB} = \mathbf{b}$  and  $\overrightarrow{XB} = 2\mathbf{c}$ ,  
then find the following in terms  
of  $a$ ,  $b$  and  $c$ :

$$\begin{aligned}\overrightarrow{OB} &= ? \\ \overrightarrow{OY} &= ? \\ \overrightarrow{AX} &= ? \\ \overrightarrow{XO} &= ? \\ \overrightarrow{YX} &= ?\end{aligned}$$

**Note:** to go in the opposite direction we need to subtract!

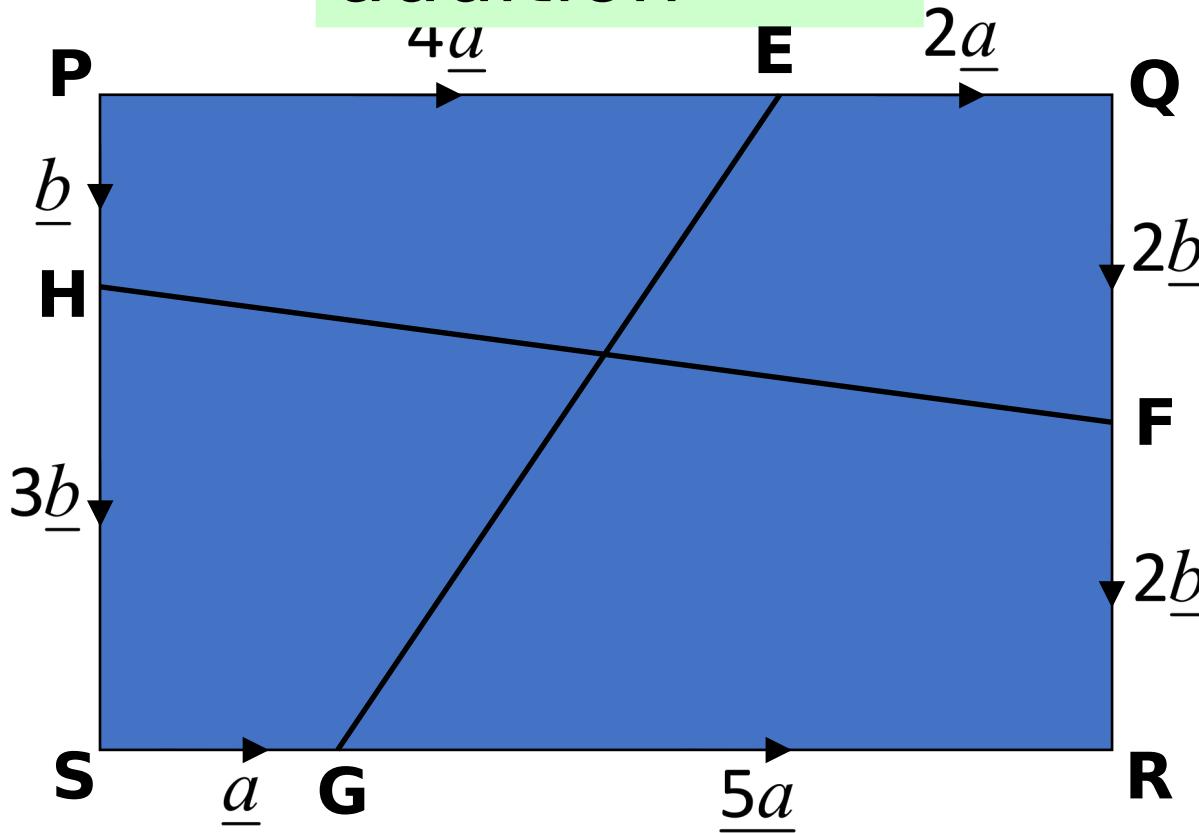
# Vector addition is commutative



Like normal addition, order doesn't matter when adding vectors

$$\underline{a} + \underline{b} = \underline{b} + \underline{a}$$

# Vector addition



On your mini white boards, write down vectors, in terms of  $\vec{a}$  and  $\vec{b}$  for:

→ for:

$\overrightarrow{EF}$

$$\overline{PR} \parallel \overline{QR}$$

$\overrightarrow{EG}$

$\overrightarrow{GE}$

$\xrightarrow{RH}$

**HR**

$\overrightarrow{sq}$

$\overrightarrow{FH}$

# Vector Geometry: problem solving

Geometric problems can be solved using the rules for adding and subtracting **vectors** and multiplying

EXAMPLE :**vectors** by a **scalar**.

1 OABC is a parallelogram.

M is the midpoint of AB.

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OC} = \mathbf{c}$$

Find in terms of  $\mathbf{a}$  and  $\mathbf{c}$ .

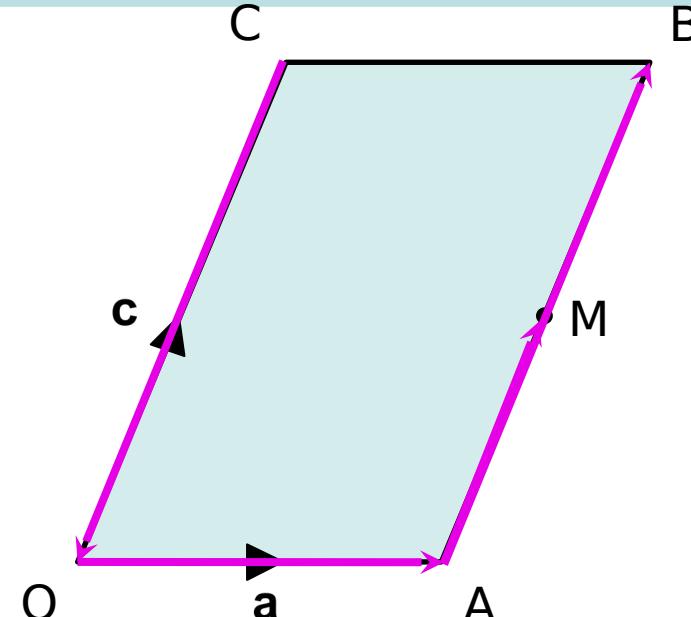
a  $\overrightarrow{CB} = \overrightarrow{OA} = \mathbf{a}$

b  $\overrightarrow{BA} = -\overrightarrow{OC} = -\mathbf{c}$

c  $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \mathbf{a} + \mathbf{c}$

d  $\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA} = -\mathbf{c} + \mathbf{a}$

e  $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} =$



Always write the journey you will take

# Vector Geometry

Geometric problems can be solved using the rules for adding and subtracting **vectors** and multiplying vectors by a **scalar**.

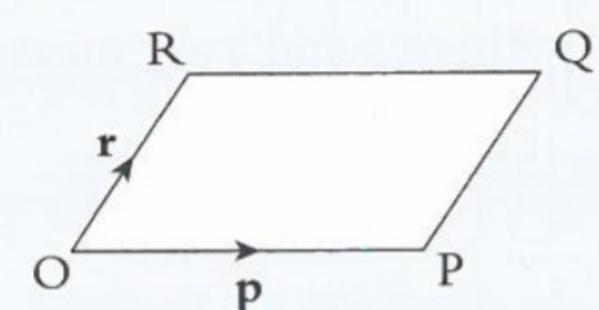
Q

1 QR is a parallelogram.

$\overrightarrow{OP} = \mathbf{p}$  and  $\overrightarrow{OR} = \mathbf{r}$ .

Find the following in terms of  $\mathbf{p}$  and  $\mathbf{r}$

- a  $\overrightarrow{RQ}$ ,
- b  $\overrightarrow{QP}$ ,
- c  $\overrightarrow{OQ}$ ,
- d  $\overrightarrow{PR}$ .



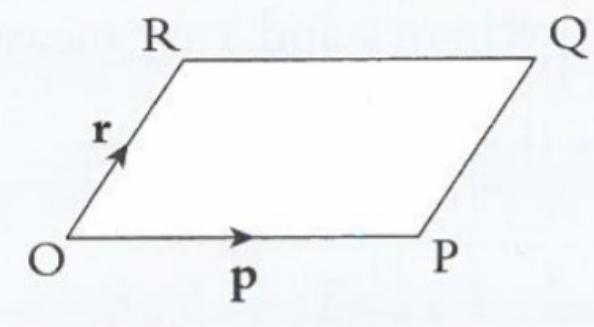
# Vector Geometry

$OPQR$  is a parallelogram.

$\vec{OP} = \mathbf{p}$  and  $\vec{OR} = \mathbf{r}$ .

Find the following in terms of  $\mathbf{p}$  and  $\mathbf{r}$

- a  $\vec{RQ}$ ,
- b  $\vec{QP}$ ,
- c  $\vec{OQ}$ ,
- d  $\vec{PR}$ .



a  $\vec{RQ} = \mathbf{p}$

b  $\vec{QP} = -\mathbf{r}$

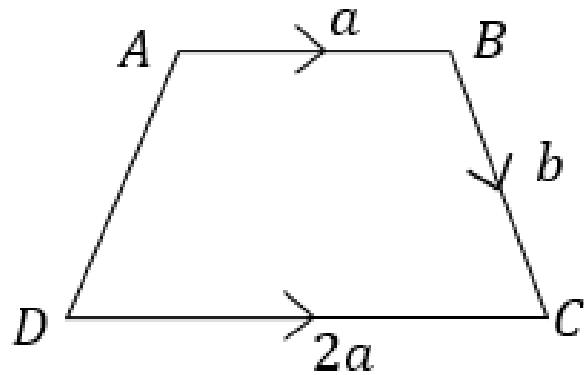
c  $\vec{OQ} = \vec{OP} + \vec{PQ} = \mathbf{p} + \mathbf{r}$

d  $\vec{PR} = \vec{PQ} + \vec{QR} = \mathbf{r} - \mathbf{p}$

Always write the journey you will take

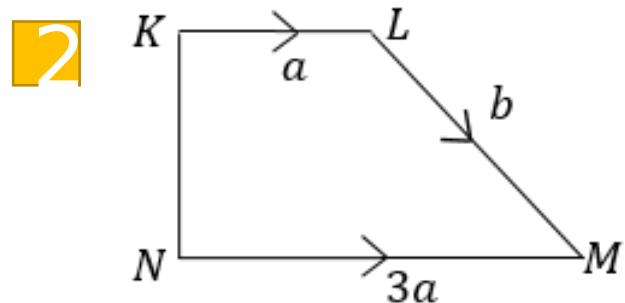
# Test your understanding

1



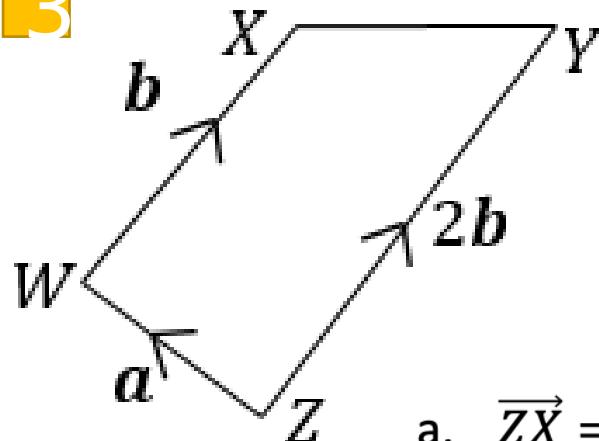
- a.  $\overrightarrow{BA} =$
- b.  $\overrightarrow{AC} =$
- c.  $\overrightarrow{DB} =$
- d.  $\overrightarrow{AD} =$

2



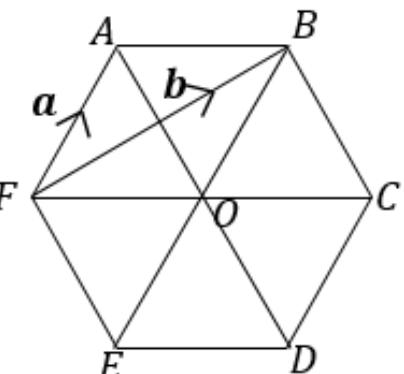
- a.  $\overrightarrow{MK} =$
- b.  $\overrightarrow{NL} =$
- c.  $\overrightarrow{NK} =$
- d.  $\overrightarrow{KN} =$

3



- a.  $\overrightarrow{ZX} =$
- b.  $\overrightarrow{YW} =$
- c.  $\overrightarrow{XY} =$
- d.  $\overrightarrow{XZ} =$

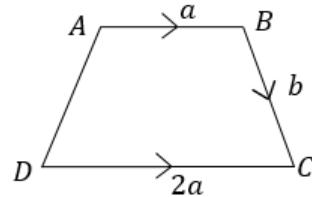
4



- a.  $\overrightarrow{AB} =$
- b.  $\overrightarrow{FO} =$
- c.  $\overrightarrow{AO} =$
- d.  $\overrightarrow{FD} =$

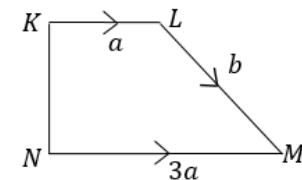
# Test your understanding

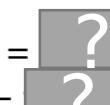
1



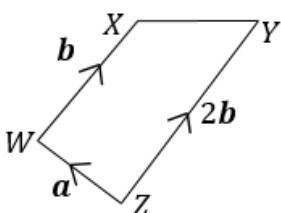
- a.  $\overrightarrow{BA} =$  
- b.  $\overrightarrow{AC} =$  
- c.  $\overrightarrow{DB} =$  
- d.  $\overrightarrow{AD} =$  

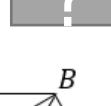
2



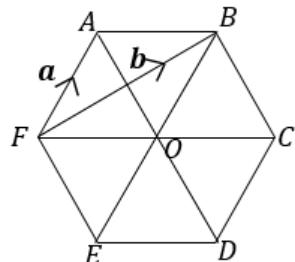
- a.  $\overrightarrow{MK} =$  
- b.  $\overrightarrow{NL} =$  
- c.  $\overrightarrow{NK} =$  
- d.  $\overrightarrow{KN} =$  

3



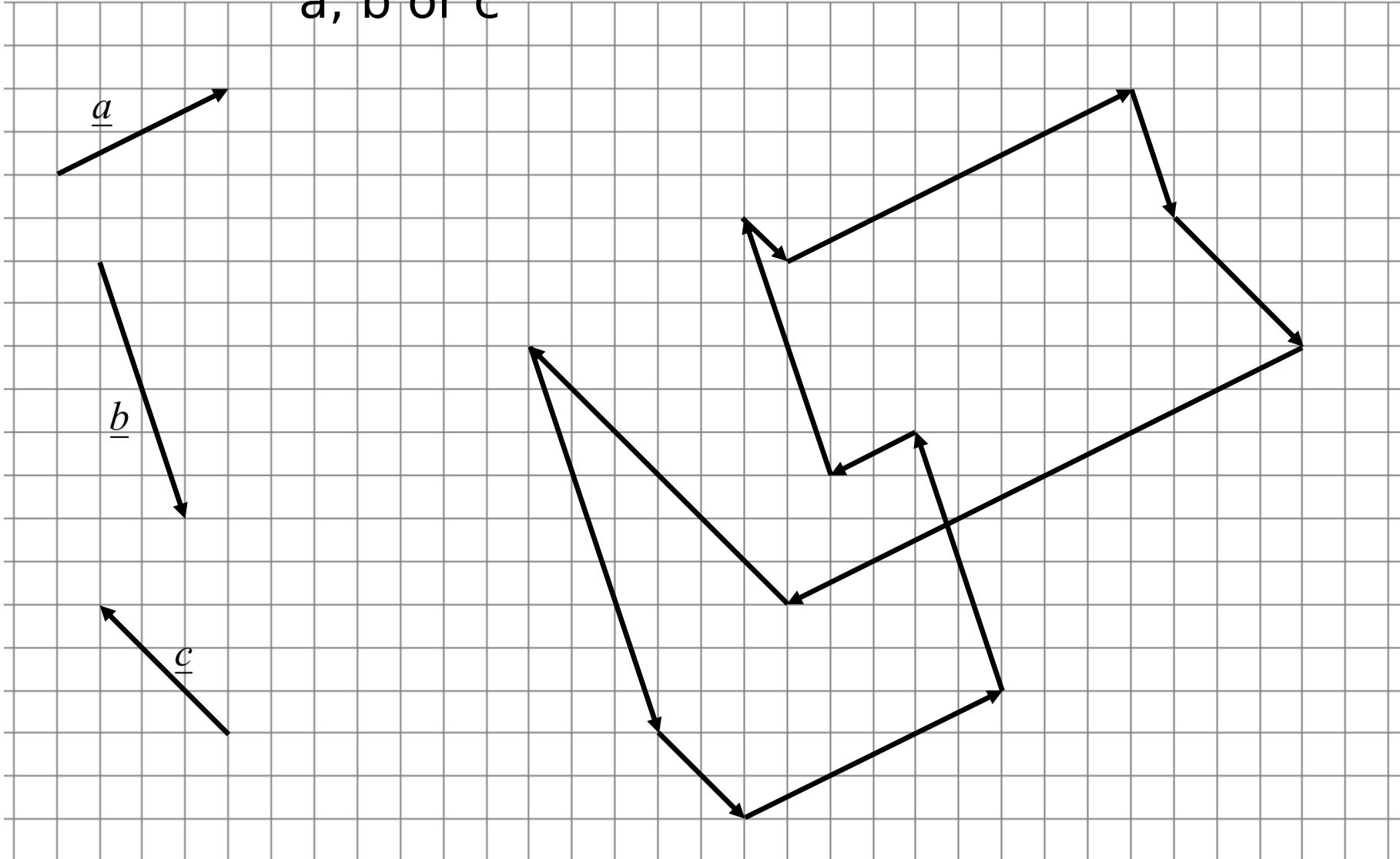
- a.  $\overrightarrow{ZX} =$  
- b.  $\overrightarrow{YW} =$  
- c.  $\overrightarrow{XY} =$  
- d.  $\overrightarrow{XZ} =$  

4



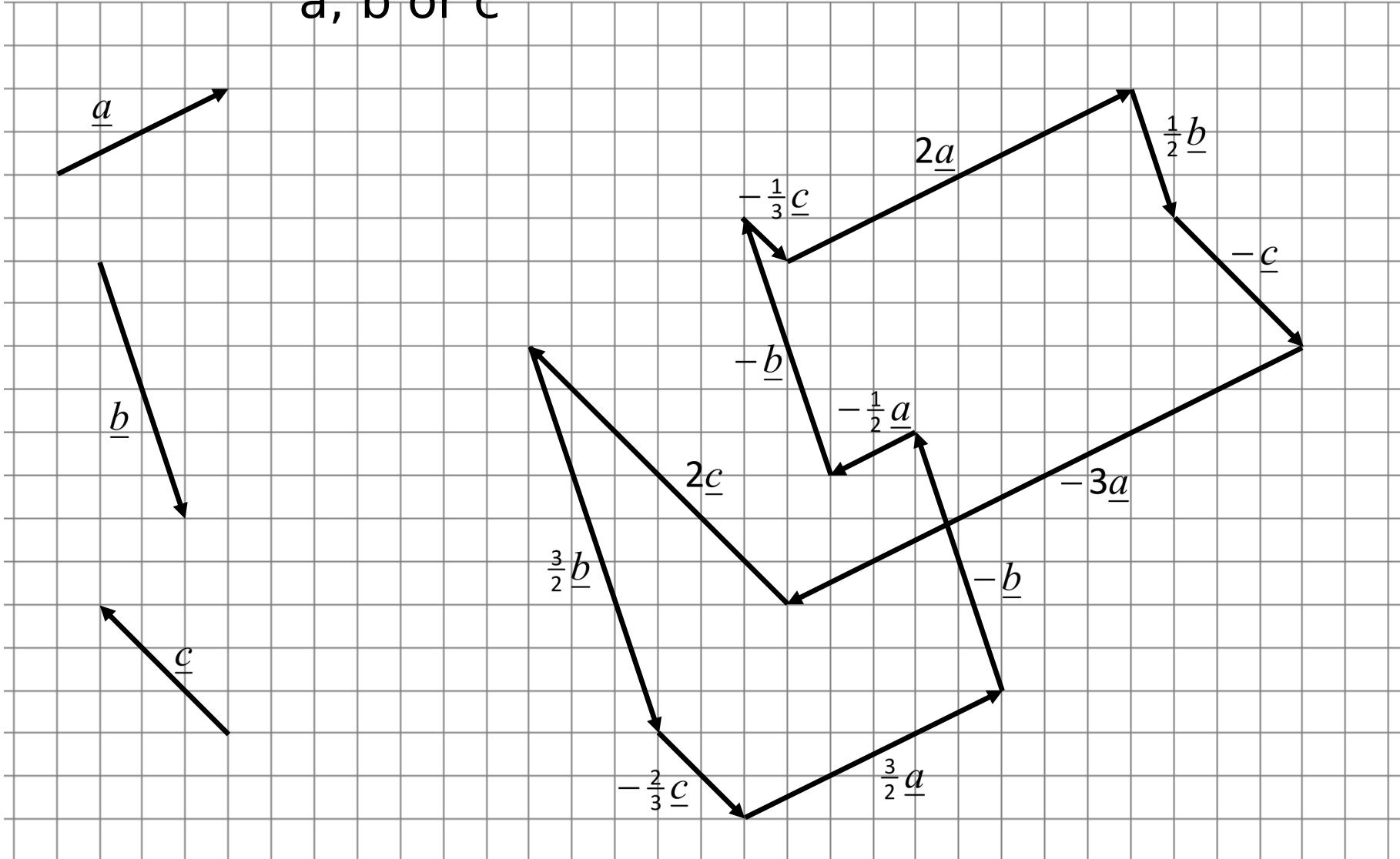
- a.  $\overrightarrow{AB} =$  
- b.  $\overrightarrow{FO} =$  
- c.  $\overrightarrow{AO} =$  
- d.  $\overrightarrow{FD} =$  

Label all vectors as multiples of  
a, b or c



Can you explain why it forms  
a loop?

Label all vectors as multiples of  
a, b or c



Can you explain why it forms  
a loop?

$$2\underline{a} - 3\underline{a} + \frac{3}{2}\underline{a} - \frac{1}{2}\underline{a} = 0$$

$$\frac{1}{2}\underline{b} + \frac{3}{2}\underline{b} - \underline{b} - \underline{b} = 0$$

$$-\underline{c} + 2\underline{c} - \frac{2}{3}\underline{c} - \frac{1}{3}\underline{c} = 0$$

# Vector Geometry - midpoints

Q  
2

P and Q are the midpoints of sides OA and OB.

$$\overrightarrow{OP} = \mathbf{p} \text{ and } \overrightarrow{OQ} = \mathbf{q}$$

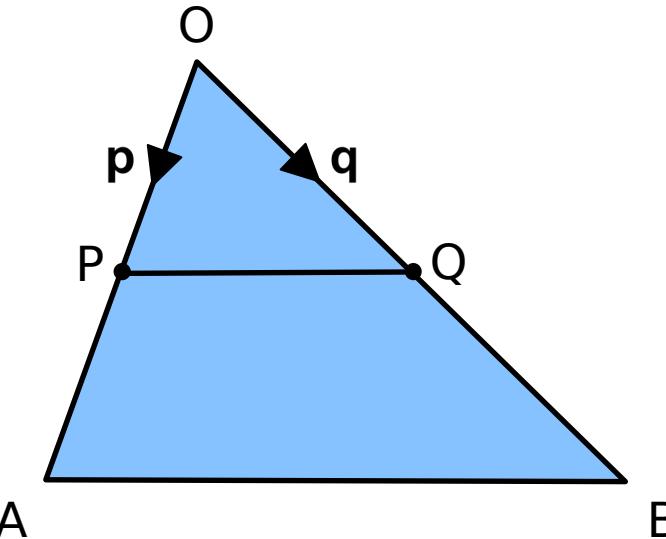
Find in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .

a  $\overrightarrow{OA}$

b  $\overrightarrow{BA}$

c  $\overrightarrow{AQ}$

d  $\overrightarrow{PB}$



Always write the journey you will take

# Vector Geometry - midpoints

Q  
2

P and Q are the midpoints of sides OA and OB.

$$\overrightarrow{OP} = \mathbf{p} \text{ and } \overrightarrow{OQ} = \mathbf{q}$$

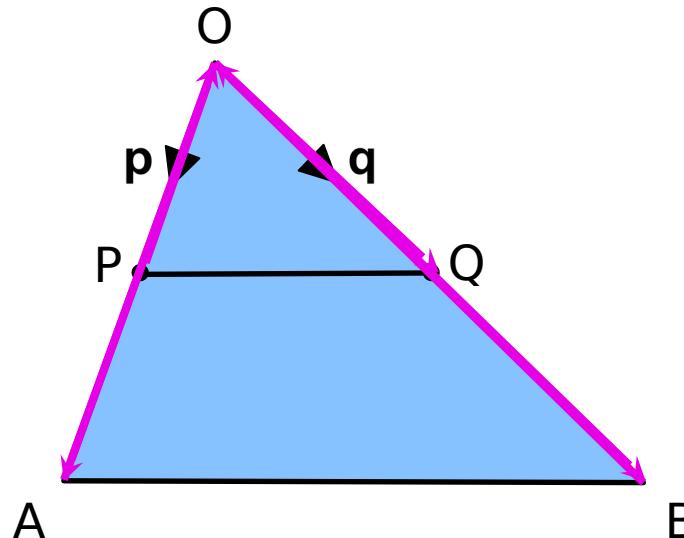
Find in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .

a  $\overrightarrow{OA} = 2\mathbf{p}$

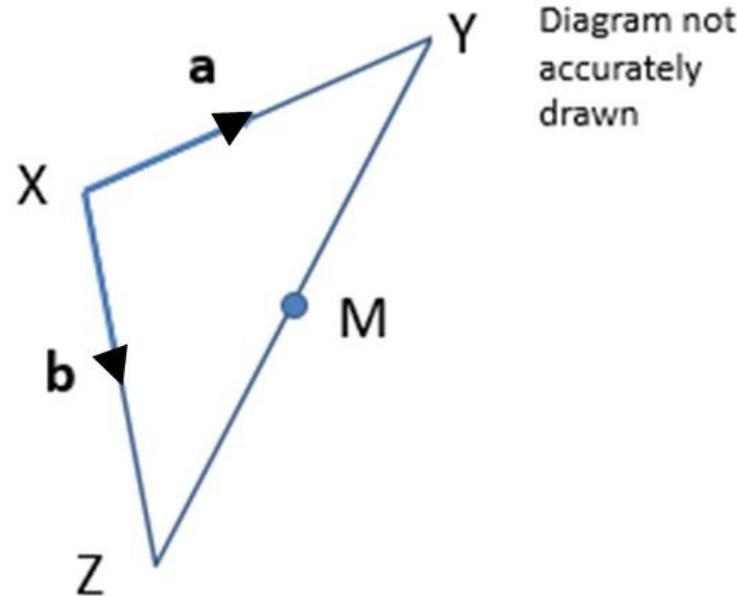
b  $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -2\mathbf{q} + 2\mathbf{p}$

c  $\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ} = -2\mathbf{p} + \mathbf{q}$

d  $\overrightarrow{PB} = \overrightarrow{PO} + \overrightarrow{OB} = -\mathbf{p} + 2\mathbf{q}$

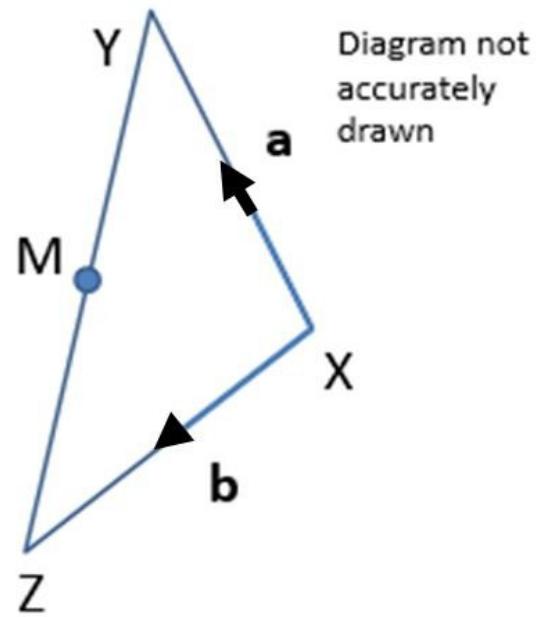


Always write the journey you will take



M is the midpoint of Y and Z

Find the vector YM



M is the midpoint of Y and Z

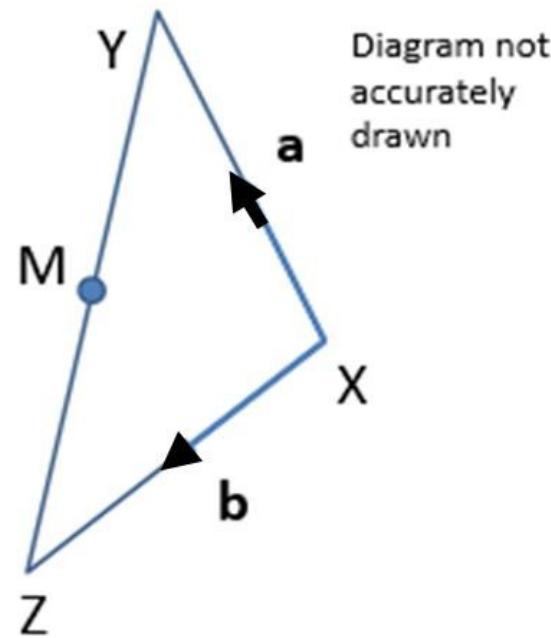
**Step 1:** Find the vector  $\overrightarrow{ZY}$

A  $-b + a$

B  $-a + b$

C  $a + b$

D  $-a - b$



M is the midpoint of Y and Z

$$\text{Vector } \overrightarrow{ZY} = -\mathbf{b} + \mathbf{a}$$

**Step 2:** Find vector  $\overrightarrow{ZM}$

A

Both B and C are correct

C

$$\frac{1}{2}(\mathbf{a} - \mathbf{b})$$

B

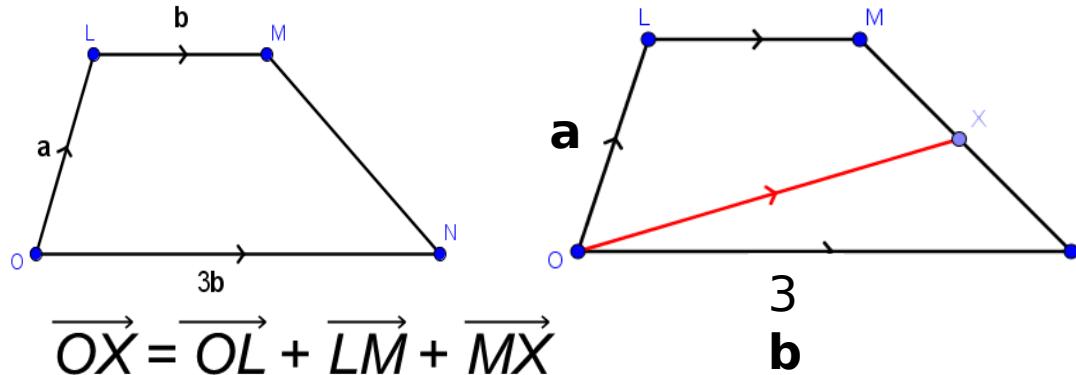
$$\frac{1}{2}(\mathbf{b} - \mathbf{a})$$

D

$$\frac{1}{2}(\mathbf{a} + \mathbf{b})$$

# TEST YOUR UNDERSTANDING

$X$  is the mid-point for the line  $MN$ . What is the position vector of  $X$ ? **b**



$$\text{but } \overrightarrow{MX} = \frac{1}{2} \overrightarrow{MN}$$

$$\begin{aligned}\overrightarrow{MN} &= -b - a + 3b \\ &= -a + 2b\end{aligned}$$

$$\begin{aligned}\overrightarrow{OX} &= a + b + \frac{1}{2}(-a + 2b) \\ &= \frac{1}{2}a + 2b\end{aligned}$$

# vector match

Which routes do these describe?

Match them to the vectors at the bottom.

$$c + e + m$$

$$m - \frac{b}{2}$$

$$\frac{b}{2}$$

$$\frac{c - b}{2}$$

$$\frac{b + c - b}{2}$$

$$\frac{c - e}{2}$$

$$b - c$$

$$c - 2m$$

$$b - 2m$$

$$\frac{b + c}{2}$$

$$m - c - b$$

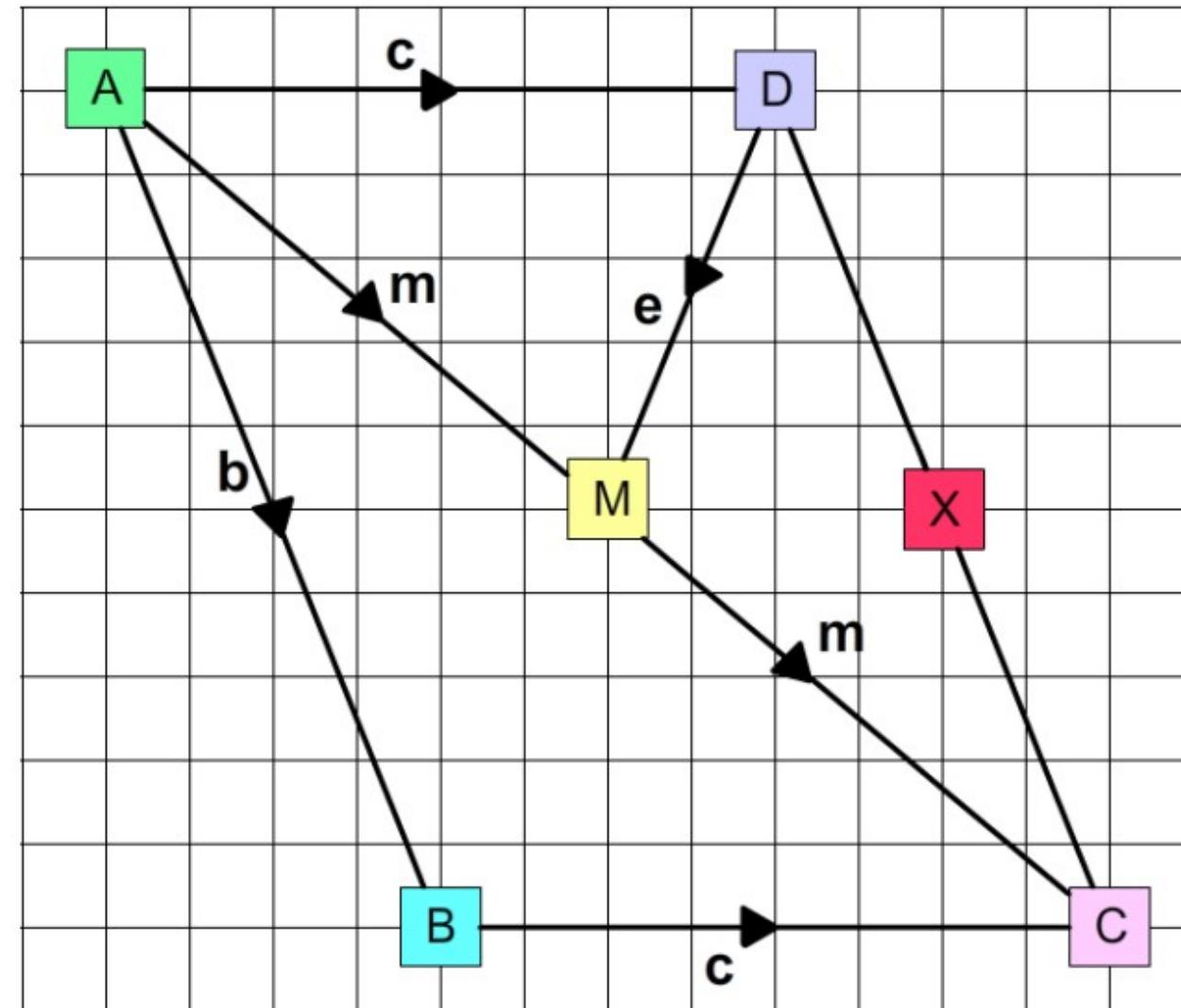
$$b + c - m - e$$

$$\frac{b - c}{2}$$

$$c + b - c$$

## Answers

BX	MX	DX	DM	BM	DB	AM
AC	AB	AD	AX	CB	MA	BA



# Answers

$$c + e + m$$

$$\xrightarrow{AC}$$

$$\frac{c - e}{2}$$

$$\xrightarrow{BX}$$

$$m - c - b$$

$$\xrightarrow{MA}$$

$$m - \frac{b}{2}$$

$$\xrightarrow{MX}$$

$$b - c$$

$$\xrightarrow{DB}$$

$$b + c - m - e$$

$$\xrightarrow{AD}$$

$$\frac{b}{2}$$

$$\xrightarrow{DX}$$

$$c - 2m$$

$$\xrightarrow{BA}$$

$$\frac{b - c}{2}$$

$$\xrightarrow{DM}$$

$$\frac{c - b}{2}$$

$$\xrightarrow{BM}$$

$$b - 2m$$

$$\xrightarrow{CB}$$

$$c + b - c$$

$$\xrightarrow{AB}$$

$$\frac{b + c - b}{2}$$

$$\xrightarrow{AX}$$

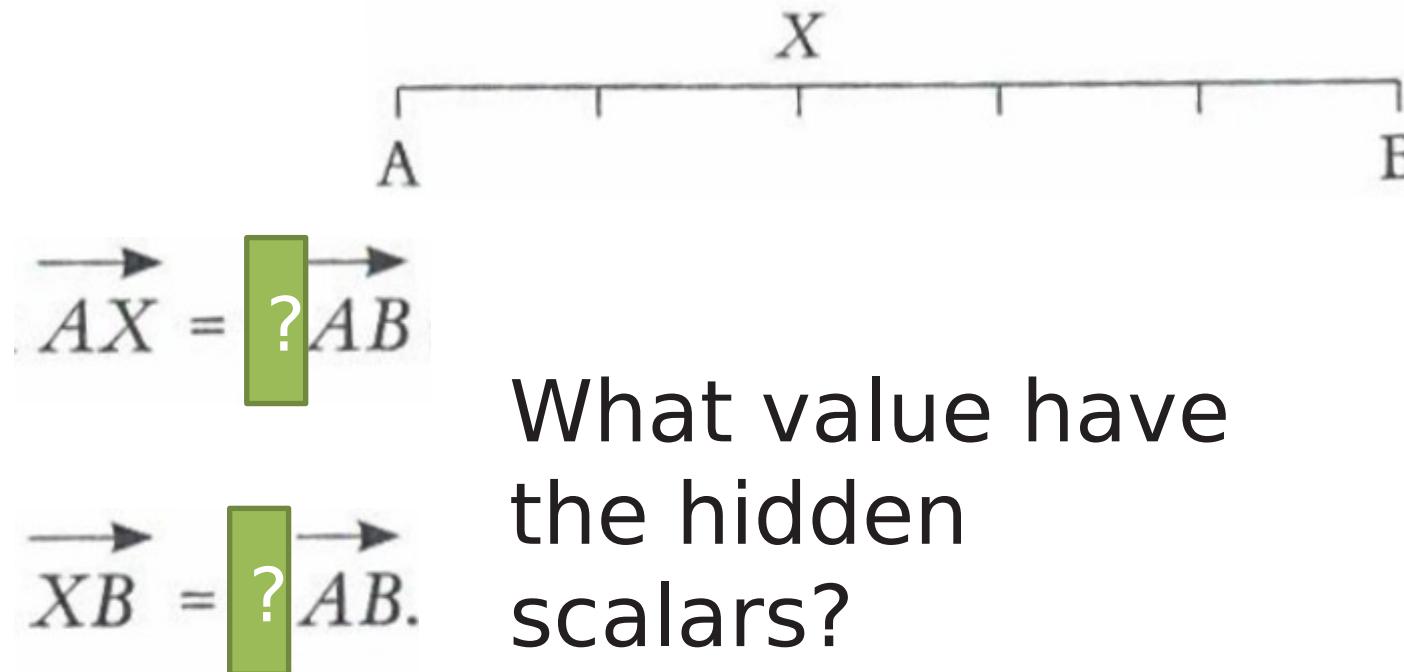
$$\frac{b + c}{2}$$

$$\xrightarrow{AM}$$

# Vector Geometry - ratios

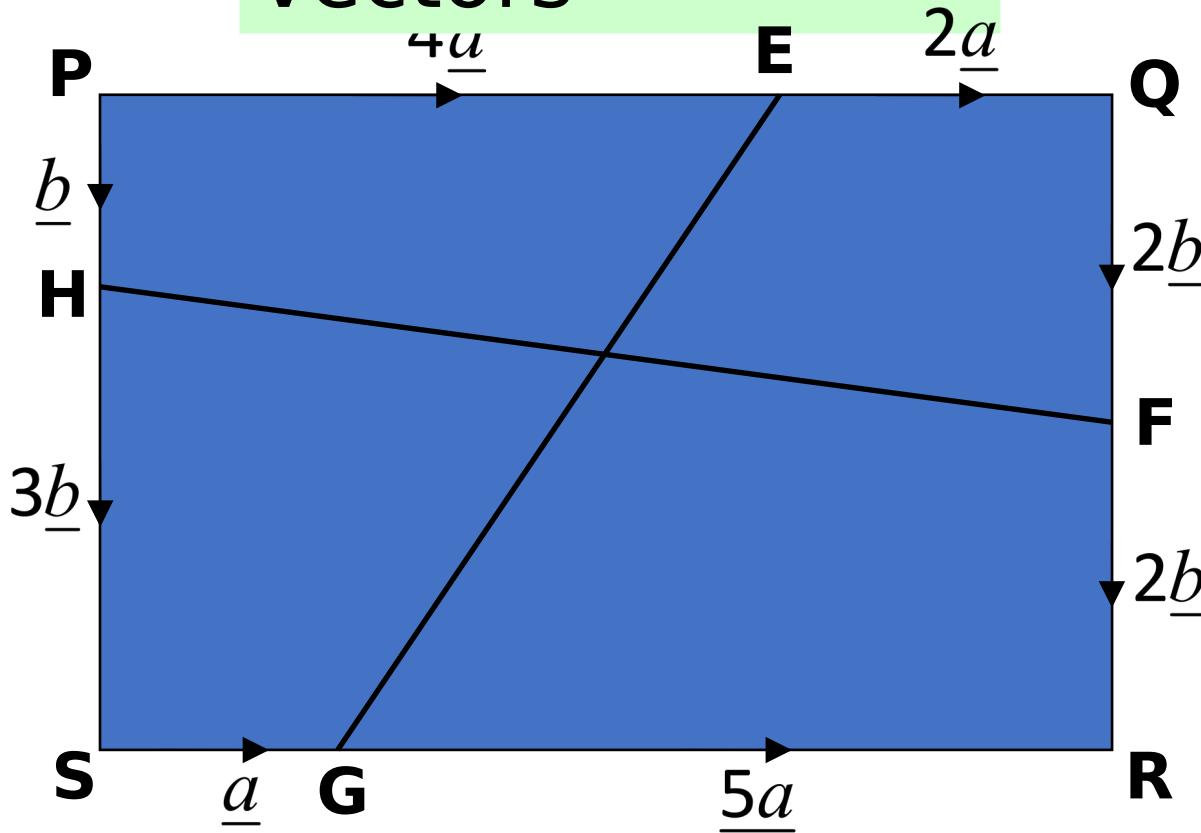
Ratios can also be used in vector geometry

If  $X$  is a point on the line  $AB$  such that  $AX : XB = 2 : 3$ , label  $X$  on the line:



What value have  
the hidden  
scalars?

# Comparing vectors



On your mini white boards, write down the **ratio of the lengths** of the lines:

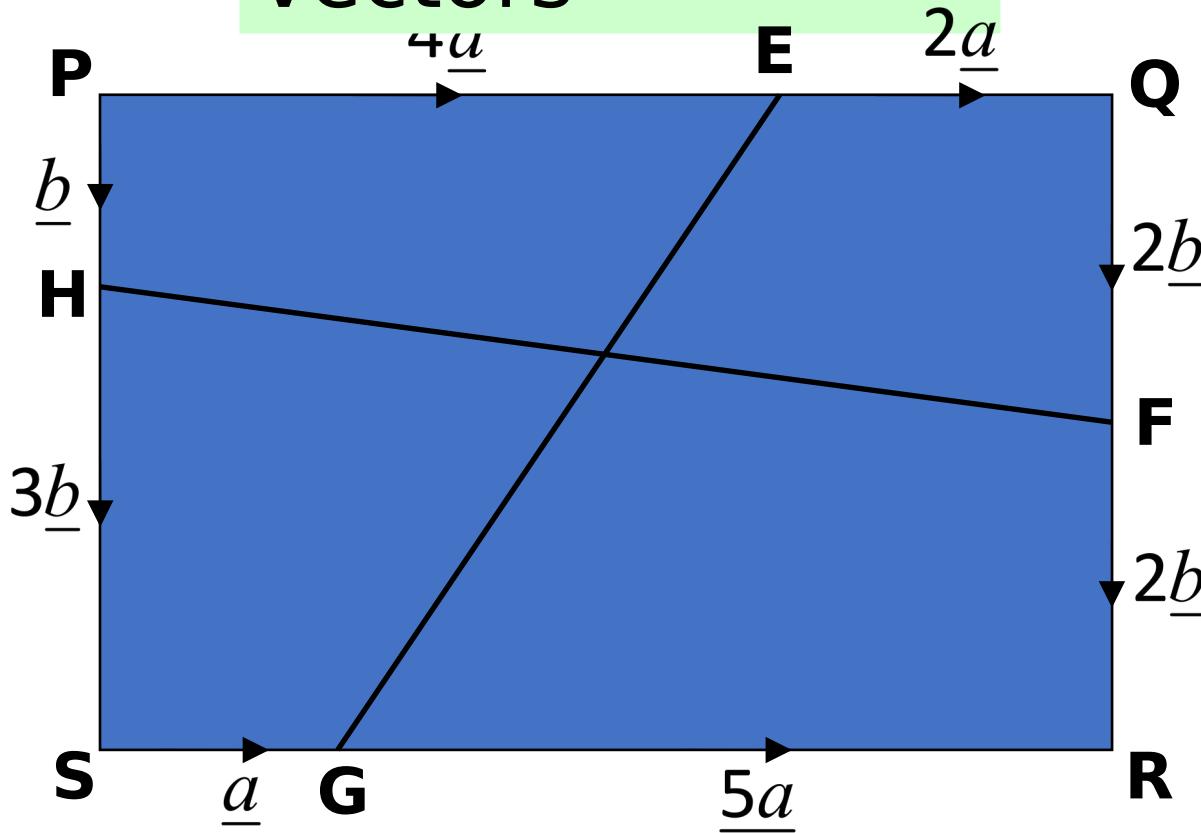
PE : EQ

SG : SR

PS : HS

QF : FR

# Comparing vectors



On your mini white boards, write down the **ratio of the lengths** of the lines:

$$PE : EQ$$

2:1

$$SG : SR$$

1:5

$$PS : HS$$

4:3

$$QF : FR$$

1:1

# Vector Geometry - ratios

## EXAMPLE

$$AP : PB = 2 : 1$$

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OB} = \mathbf{b}$$

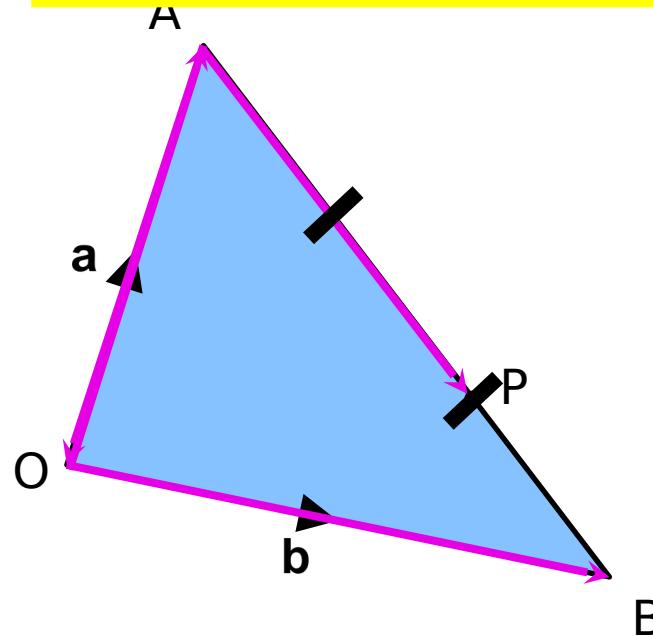
Find in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

a  $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$   
 $= -\mathbf{a} + \mathbf{b}$

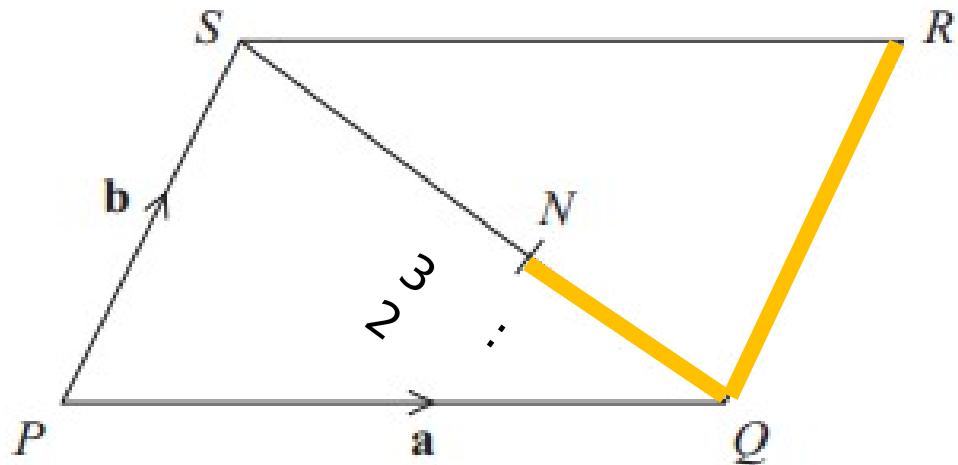
b  $\overrightarrow{AP} = \frac{2}{3} \overrightarrow{AB}$   
 $= \frac{2}{3}(-\mathbf{a} + \mathbf{b})$

c  $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$   
 $= \mathbf{a} + \frac{2}{3}(-\mathbf{a} + \mathbf{b})$   
 $= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

First label the diagram with the ratios!



# Vector Geometry - ratios



a  $\overrightarrow{SQ} = -b + a$

PQRS is a parallelogram.

N is the point on  $SQ$  such that  $SN : NQ = 3 : 2$

$$\overrightarrow{PQ} = \mathbf{a} \quad \overrightarrow{PS} = \mathbf{b}$$

(a) Write down, in terms of **a** and **b**, an expression for  $\overrightarrow{SQ}$ .

(b) Express  $\overrightarrow{NR}$  in terms of **a** and **b**.

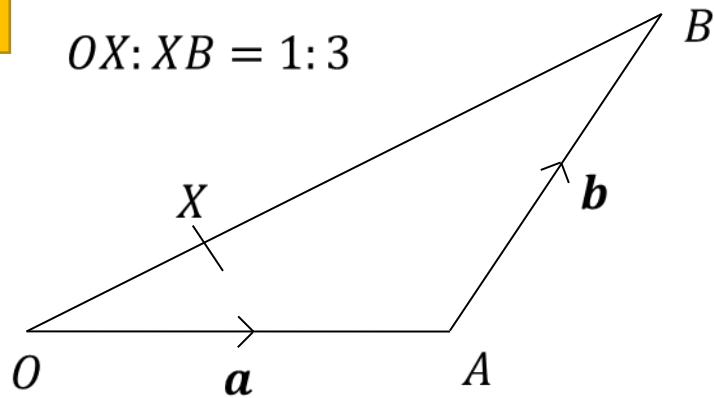
b

?

# Test your understanding

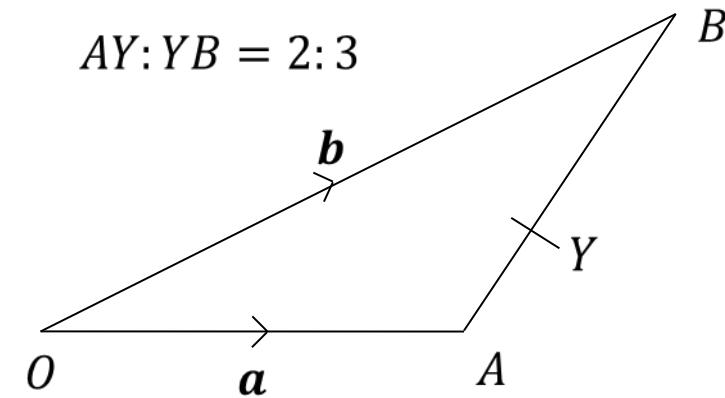
A

$$OX:XB = 1:3$$



B

$$AY:YB = 2:3$$



$$\overrightarrow{AX} =$$

First Step?

=

=

=

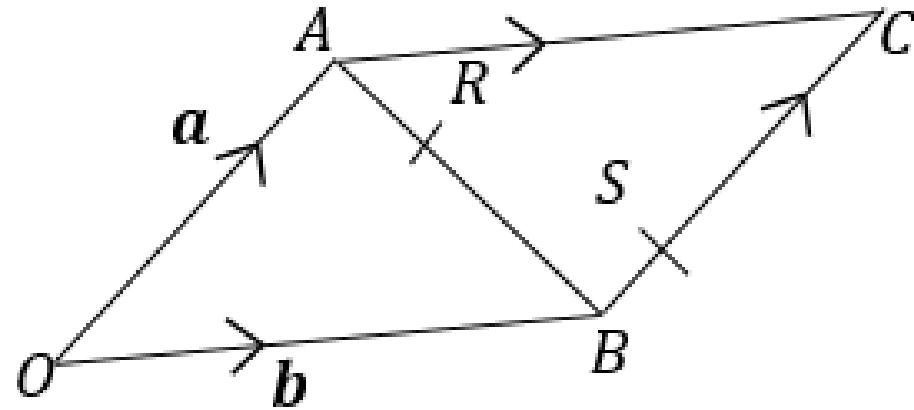
?

$$\overrightarrow{OY} =$$

First Step?

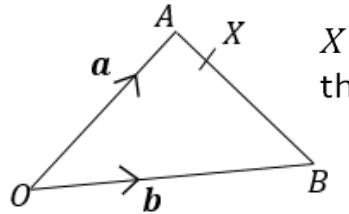
?

$OACB$  is a parallelogram.  $R$  is a point such that  $AR: RB = 2: 3$ .  
 $S$  is a point such that  $BS: SC = 1: 3$ .



- a.  $\overrightarrow{OR} =$
- b.  $\overrightarrow{BS} =$
- c.  $\overrightarrow{OS} =$
- d.  $\overrightarrow{RS} =$

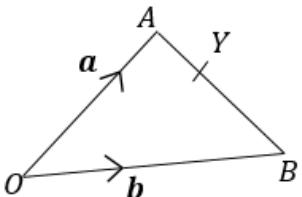
1



$X$  is a point such that  $AX:XB = 1:4$

- a.  $\overrightarrow{AB} =$
- b.  $\overrightarrow{AX} =$
- c.  $\overrightarrow{OX} =$
- d.  $\overrightarrow{BX} =$

2



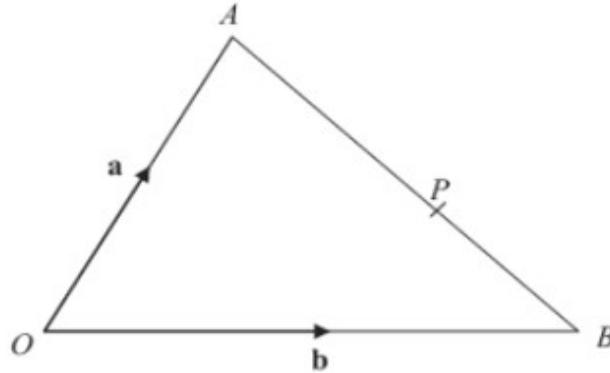
$Y$  is a point such that  $YB = 2AY$

a.  $\overrightarrow{AY} =$

b.  $\overrightarrow{OY} =$

c.  $\overrightarrow{YO} =$

3



a) Find  $\overrightarrow{AB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

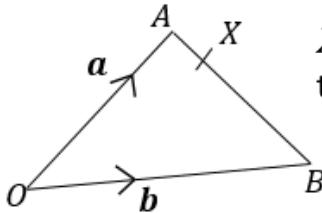
$$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b} \text{ or } \mathbf{b} - \mathbf{a}$$

b)  $P$  is on  $AB$  such that  $AP:PB = 3:2$ .

Show that  $\overrightarrow{OP} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$

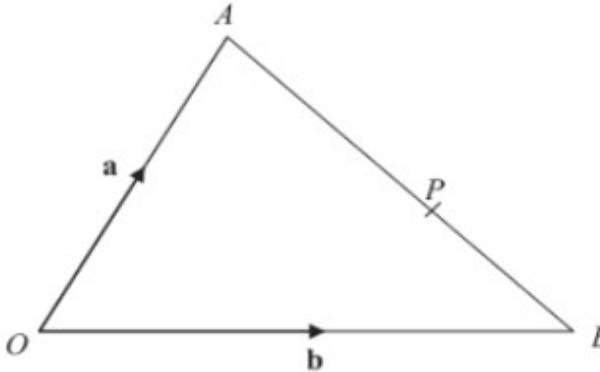
# Quick practice

1

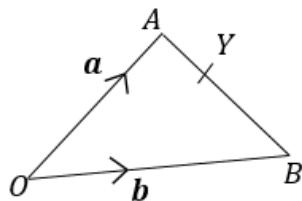


X is a point such that  $AX:XB = 1:4$

3



2



Y is a point such that  $YB = 2AY$

- a.  $\overrightarrow{AY} =$
- b.  $\overrightarrow{OY} =$
- c.  $\overrightarrow{YO} =$

a) Find  $\overrightarrow{AB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

b) P is on AB such that  $AP:PB = 3:2$ .

Show that  $\overrightarrow{OP} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$

# Exam style question

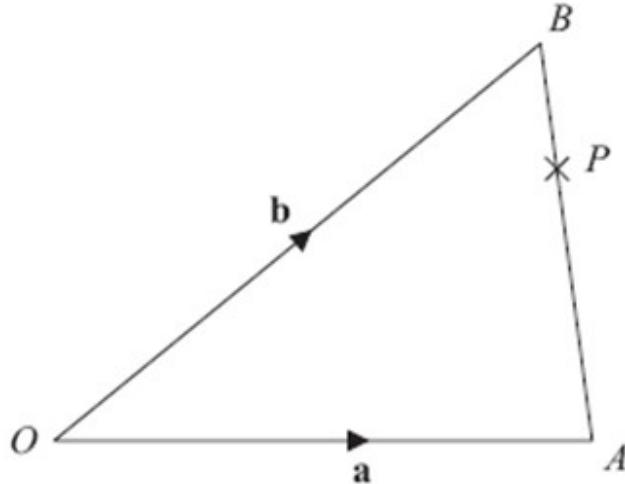


Diagram NOT  
accurately drawn

$OAB$  is a triangle.

$$\overrightarrow{OA} = \mathbf{a}$$

$$\overrightarrow{OB} = \mathbf{b}$$

(a) Find  $\overrightarrow{AB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

?

(1)

$P$  is the point on  $AB$  such that  $AP : PB = 3 : 1$

(b) Find  $\overrightarrow{OP}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .  
Give your answer in its simplest form.

?

You MUST  
expand  
and  
simplify.

# All together

Q  
5

OPQR is a trapezium.

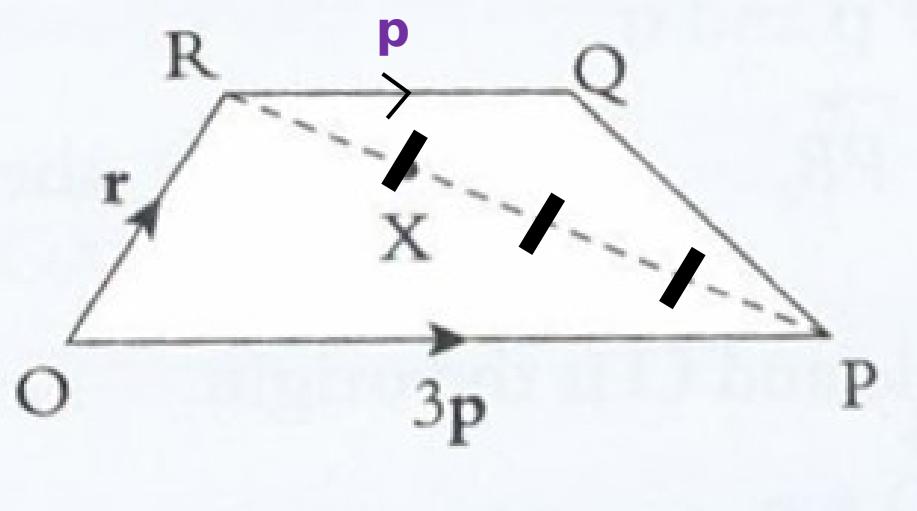
$$\overrightarrow{OP} = 3\mathbf{p} \text{ and } \overrightarrow{OR} = \mathbf{r}$$

$$\overrightarrow{RQ} = \frac{1}{3} \overrightarrow{OP}$$

X lies on RP such that  $RX : XP = 1 : 3$

► **NOTE:**  $RX : XP = 1 : 3$  means that  $\overrightarrow{RX} = \frac{1}{4} \overrightarrow{RP}$ .

First label the diagram with the ratios!



a Find in terms of  $\mathbf{p}$  and  $\mathbf{r}$

- i  $\overrightarrow{OQ}$ ,
- ii  $\overrightarrow{RP}$ ,
- iii  $\overrightarrow{RX}$ ,
- iv  $\overrightarrow{OX}$ .

b What do your answers for  $\overrightarrow{OQ}$  and  $\overrightarrow{OX}$  tell you about the points O, X and Q?

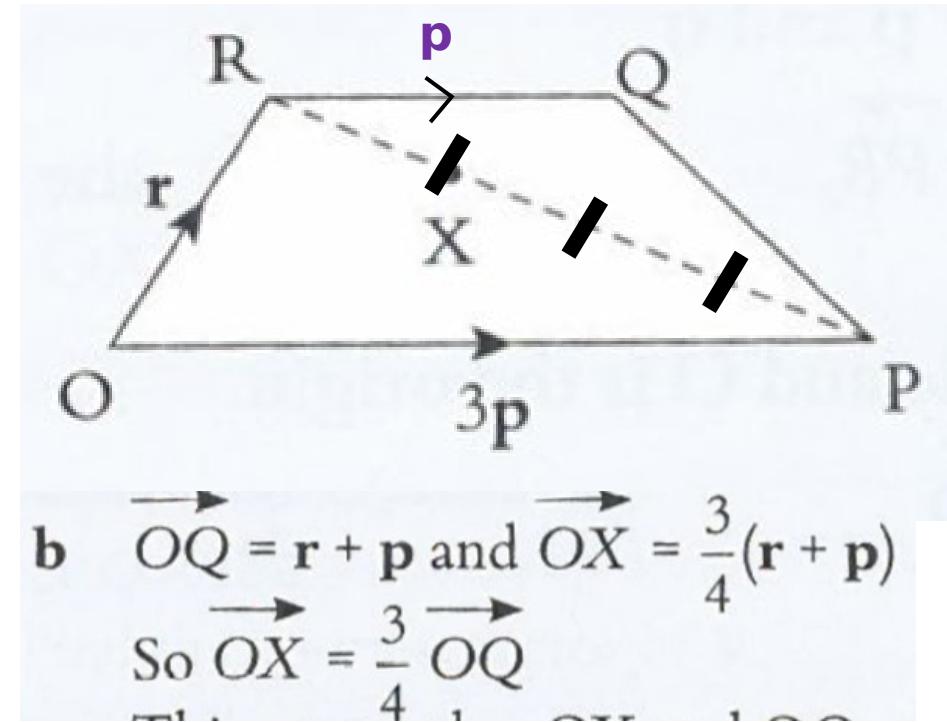
# SOLUTION

a i  $\overrightarrow{OQ} = \overrightarrow{OR} + \overrightarrow{RQ}$   
 $= \mathbf{r} + \mathbf{p}$

ii  $\overrightarrow{RP} = \overrightarrow{RO} + \overrightarrow{OP}$   
 $= -\mathbf{r} + 3\mathbf{p}$

iii  $\overrightarrow{RX} = \frac{1}{4} \overrightarrow{RP}$   
 $= \frac{1}{4}(-\mathbf{r} + 3\mathbf{p})$   
 $= \frac{3}{4}\mathbf{p} - \frac{1}{4}\mathbf{r}$

iv  $\overrightarrow{OX} = \overrightarrow{OR} + \overrightarrow{RX} = \mathbf{r} + \frac{3}{4}\mathbf{p} - \frac{1}{4}\mathbf{r}$   
 $= \frac{3}{4}\mathbf{r} + \frac{3}{4}\mathbf{p}$   
 $= \frac{3}{4}(\mathbf{r} + \mathbf{p})$



This means that OX and OQ are **parallel**.  
The points O lies on both of these line segments.  
So the points O, X and Q are **collinear**.

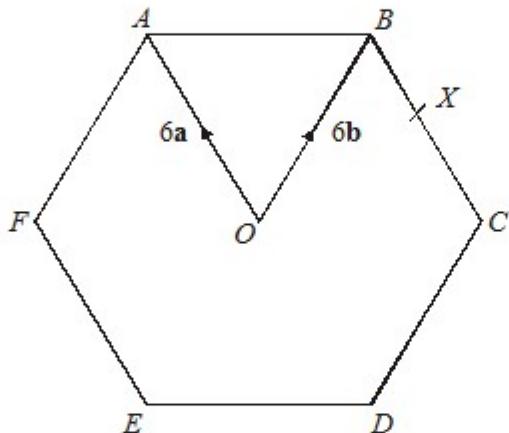


Diagram **NOT**  
accurately drawn

The diagram shows a regular hexagon  $ABCDEF$  with centre  $O$ .

$$\overrightarrow{OA} = 6\mathbf{a} \quad \overrightarrow{OB} = 6\mathbf{b}$$

(a) Express in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$

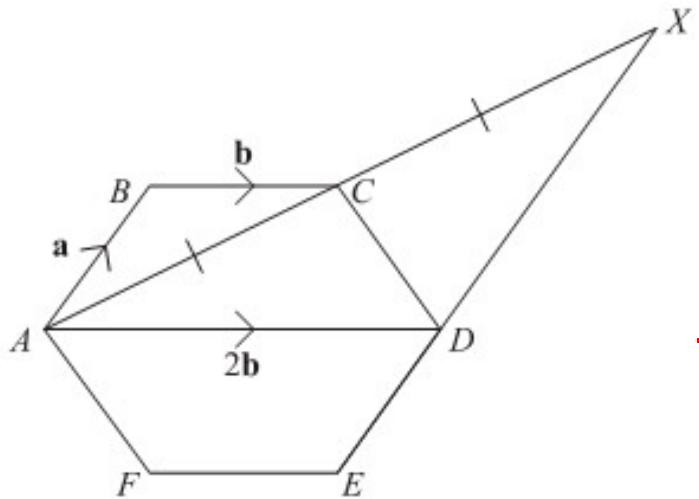
(i)  $\overrightarrow{AB}$ ,

.....

(ii)  $\overrightarrow{EF}$ .

$X$  is the midpoint of  $BC$ .

(b) Express  $\overrightarrow{EX}$  in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$



$$\begin{matrix} DX \\ -2b + a + b \\ a + b \\ 2a \end{matrix}$$

Diagram **NOT**  
accurately drawn

$ABCDEF$  is a regular hexagon.

$$\overrightarrow{AB} = \mathbf{a} \quad \overrightarrow{BC} = \mathbf{b} \quad \overrightarrow{AD} = 2\mathbf{b}$$

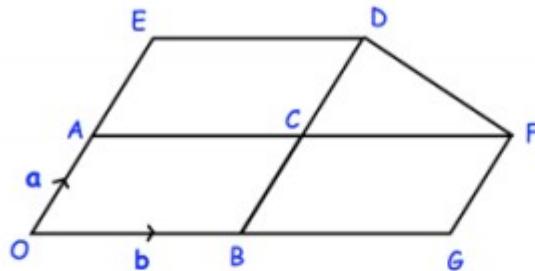
(a) Find the vector  $\overrightarrow{AC}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .  $\mathbf{a} + \mathbf{b}$

$$\overrightarrow{AC} = \overrightarrow{CX}$$

(b) Prove that  $AB$  is parallel to  $DX$ .

1. In the diagram OBDE and OAFG are parallelograms.  
 B is the midpoint of OG.  
 A is the midpoint of OE.

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OB} = \mathbf{b}$$



(a) Express, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , the following vectors.  
 Give your answers in their simplest form.

(i)  $\overrightarrow{OC}$

$\mathbf{a} + \mathbf{b}$

(1)

(ii)  $\overrightarrow{BA}$

$\mathbf{a} - \mathbf{b}$

(1)

(iii)  $\overrightarrow{DF}$

$\mathbf{b} - \mathbf{a}$

(1)

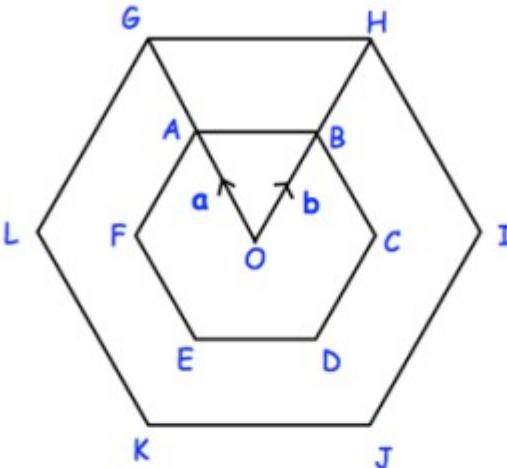
(b) Show  $\overrightarrow{EG}$  and  $\overrightarrow{DF}$  are parallel.

$$\begin{aligned}\overrightarrow{EG} &= 2\mathbf{b} - 2\mathbf{a} \\ \overrightarrow{DF} &= \mathbf{b} - \mathbf{a}\end{aligned}$$

$$\begin{aligned}\overrightarrow{EG} &= 2\overrightarrow{DF} \\ \therefore \text{they are parallel}\end{aligned}$$

(2)

2.



ABCDEF and GHIJKL are regular hexagons with centre O.  
GHIJKL is an enlargement of ABCDEF, with scale factor 2.

$$\overrightarrow{OA} = \mathbf{a} \quad \text{and} \quad \overrightarrow{OB} = \mathbf{b}$$

(a) Write the vector  $\overrightarrow{AB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{\mathbf{b} - \mathbf{a}}$$

(1)

(b) Write the vector  $\overrightarrow{OG}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{2\mathbf{a}}$$

(1)

(c) Write the vector  $\overrightarrow{OE}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{-\mathbf{b}}$$

(1)

(d) Write the vector  $\overrightarrow{FC}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{2\mathbf{b} - 2\mathbf{a}}$$

(1)

(e) Write the vector  $\overrightarrow{IK}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{2\mathbf{a} - 4\mathbf{b}}$$

(1)

(f) Write the vector  $\overrightarrow{LI}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{4\mathbf{b} - 4\mathbf{a}}$$

(1)

(g) Write the vector  $\overrightarrow{LG}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{2\mathbf{b}}$$

(1)

(h) Write the vector  $\overrightarrow{JG}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{4\mathbf{b}}$$

(1)

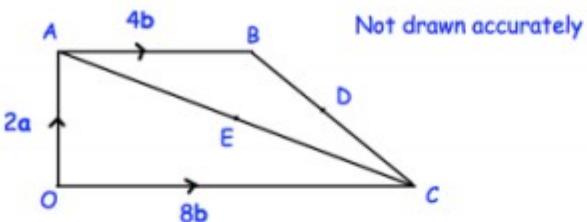
(i) Write the vector  $\overrightarrow{DL}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\underline{3\mathbf{a} - 2\mathbf{b}}$$

(1)

3. OABC is a trapezium.  
 Point D is the midpoint of BC.  
 Point E is the midpoint of AC.

$$\vec{OA} = 2\mathbf{a} \quad \vec{AB} = 4\mathbf{b} \quad \text{and} \quad \vec{OC} = 8\mathbf{b}$$



(a) Write these vectors in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

(i)  $\vec{OB}$

$$2\mathbf{a} + 4\mathbf{b}$$

(1)

(ii)  $\vec{AC}$

$$8\mathbf{b} - 2\mathbf{a}$$

(1)

(iii)  $\vec{AE}$

$$4\mathbf{b} - \mathbf{a}$$

(1)

(b) Show  $\vec{ED}$  and  $\vec{OC}$  are parallel.

$$\vec{OC} = 8\mathbf{b}$$

$$\vec{ED} = \mathbf{a} - 4\mathbf{b} + 4\mathbf{b} + \frac{1}{2}(-4\mathbf{b} \cdot 2\mathbf{a} + 8\mathbf{b})$$

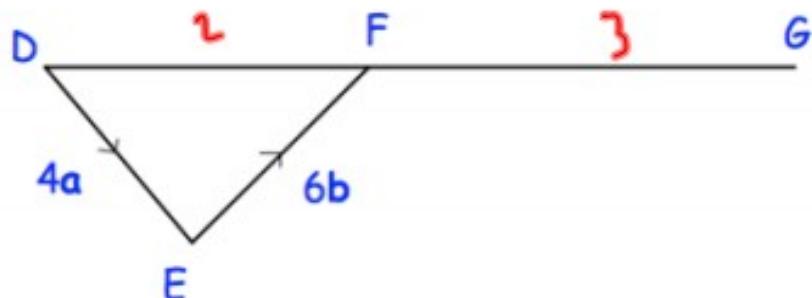
$$\vec{ED} = 8\mathbf{b} + 4\mathbf{b} - 2\mathbf{b} \cdot \mathbf{a} + 4\mathbf{b}$$

(3)

$$\vec{ED} = 2\mathbf{b} \quad \vec{OC} = 4\vec{ED} \quad \therefore \text{parallel}$$

4. DFG is a straight line.

$$\overrightarrow{DE} = 4\mathbf{a} \text{ and } \overrightarrow{EF} = 6\mathbf{b}$$



(a) Write down the vector  $\overrightarrow{DF}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$

$$4\mathbf{a} + 6\mathbf{b}$$

(1)

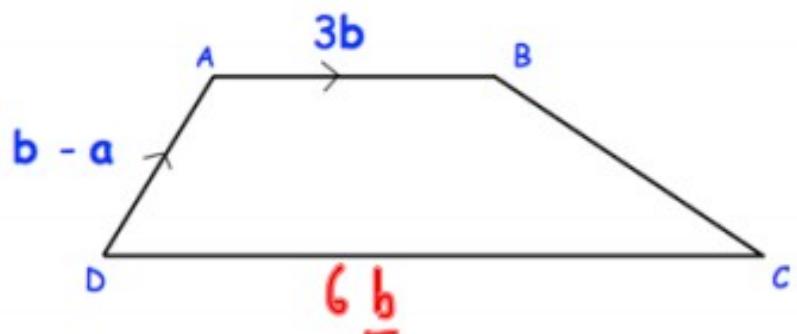
(b)  $DF : FG = 2:3$

Work out the vector  $\overrightarrow{DG}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$   
Give your answer in its simplest form.

$$(2\mathbf{a} + 3\mathbf{b}) \times 5$$
$$10\mathbf{a} + 15\mathbf{b}$$

(2)

5. ABCD is a trapezium



AB and DC are parallel.

$$DC = 2AB$$

(a) Write down the vector  $\overrightarrow{DC}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$

$$6\underline{b}$$

(1)

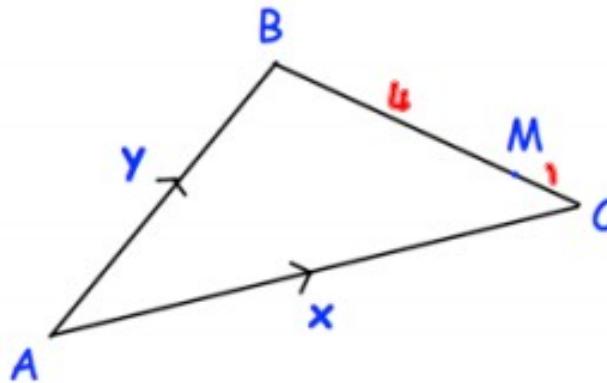
(b) Work out the vector  $\overrightarrow{BC}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$   
Give your answer in its simplest form.

$$-3\underline{b} - \underline{b} + \underline{a} + 6\underline{b}$$

$$2\underline{b} + \underline{a}$$

(2)

6.



ABC is a triangle.

M lies on BC such that  $BM = \frac{4}{5} BC$ Express these vectors in terms of  $x$  and  $y$ 

(a)  $\overrightarrow{BC}$

$$-\underline{y} + \underline{x}$$

(1)

(b)  $\overrightarrow{BM}$

$$-\frac{4}{5}\underline{y} + \frac{4}{5}\underline{x}$$

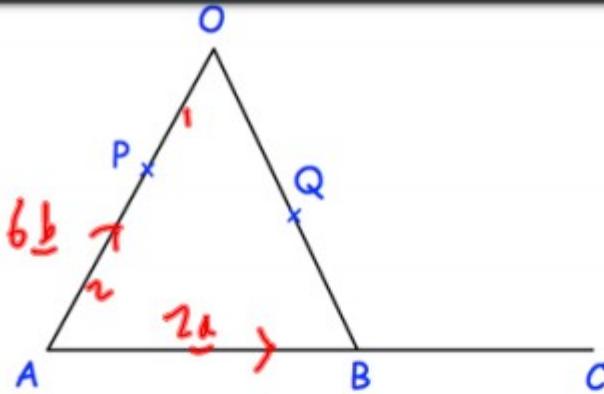
(1)

(c)  $\overrightarrow{AM}$

$$\underline{y} - \frac{4}{5}\underline{y} + \frac{4}{5}\underline{x}$$

$$\frac{1}{5}\underline{y} + \frac{4}{5}\underline{x}$$

(1)



AOB is a triangle.  
P is a point on AO.

$$\overrightarrow{AB} = 2a$$

$$\overrightarrow{AO} = 6b$$

$$AP:PO = 2:1$$

(a) Find the vector  $\overrightarrow{OB}$  in terms of  $a$  and  $b$

$$2a - 6b$$

(1)

Q is the midpoint of OB.  
B is the midpoint of AC.

(b) Show PQC is a straight line.

$$\overrightarrow{PQ} = 2b + a - 3b$$

$$\overrightarrow{PQ} = a - b$$

$$\overrightarrow{QC} = 3\overrightarrow{PQ}$$

$$\overrightarrow{QC} = 0 - 3b + 2a$$

$$\overrightarrow{QC} = 3a - 3b$$

QC and PQ are parallel and also  
both pass through the point Q.  
therefore PQC must be a  
straight line. (co-linear) (3)

4.

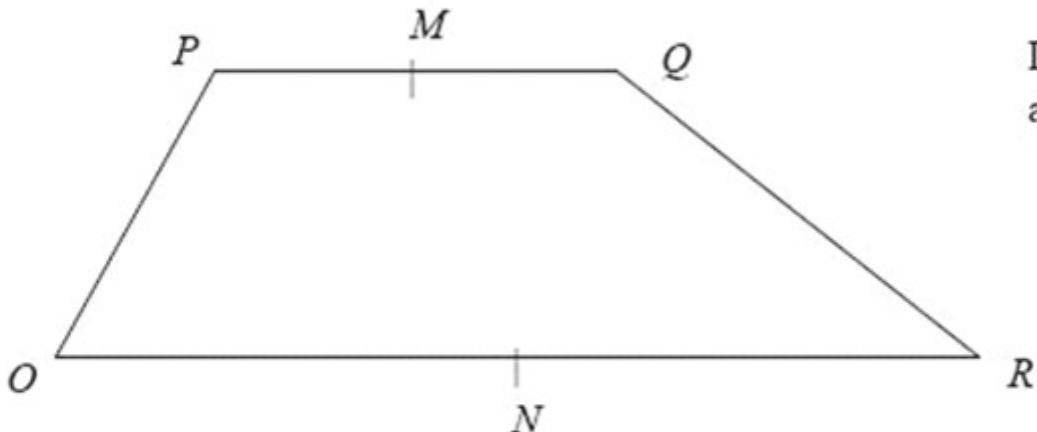


Diagram NOT  
accurately drawn

$OPQR$  is a trapezium with  $PQ$  parallel to  $OR$ .

$$\overrightarrow{OP} = 2\mathbf{b} \quad \overrightarrow{PQ} = 2\mathbf{a} \quad \overrightarrow{OR} = 6\mathbf{a}$$

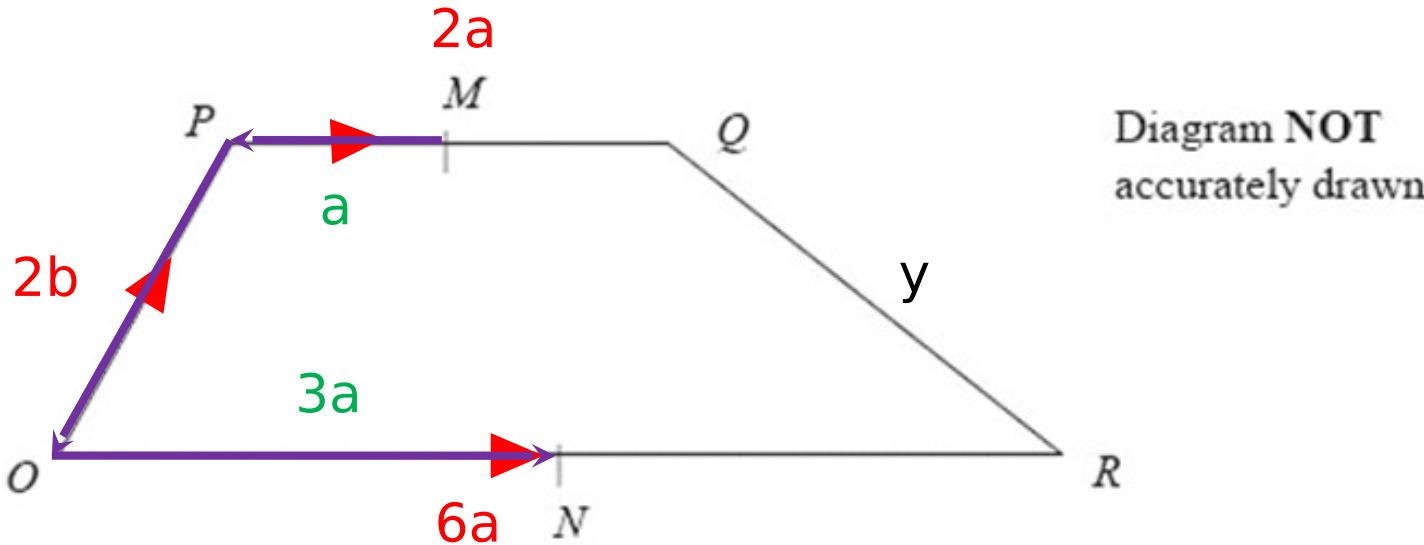
$M$  is the midpoint of  $PQ$  and  $N$  is the midpoint of  $OR$ .

(a) Find the vector  $\overrightarrow{MN}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$X$  is the midpoint of  $MN$  and  $Y$  is the midpoint of  $QR$ .

(b) Prove that  $XY$  is parallel to  $OR$ .

4.



$OPQR$  is a trapezium with  $PQ$  parallel to  $OR$ .

$$\overrightarrow{OP} = 2\mathbf{b}$$

$$\overrightarrow{PQ} = 2\mathbf{a}$$

$$\overrightarrow{OR} = 6\mathbf{a}$$

$$\begin{matrix} -\mathbf{a} & -2\mathbf{b} & +3\mathbf{a} \\ 2\mathbf{a} & -2\mathbf{b} \end{matrix}$$

$M$  is the midpoint of  $PQ$  and  $N$  is the midpoint of  $OR$ .

(a) Find the vector  $\overrightarrow{MN}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$X$  is the midpoint of  $MN$  and  $Y$  is the midpoint of  $QR$ .

(b) Prove that  $XY$  is parallel to  $OR$ .

4.

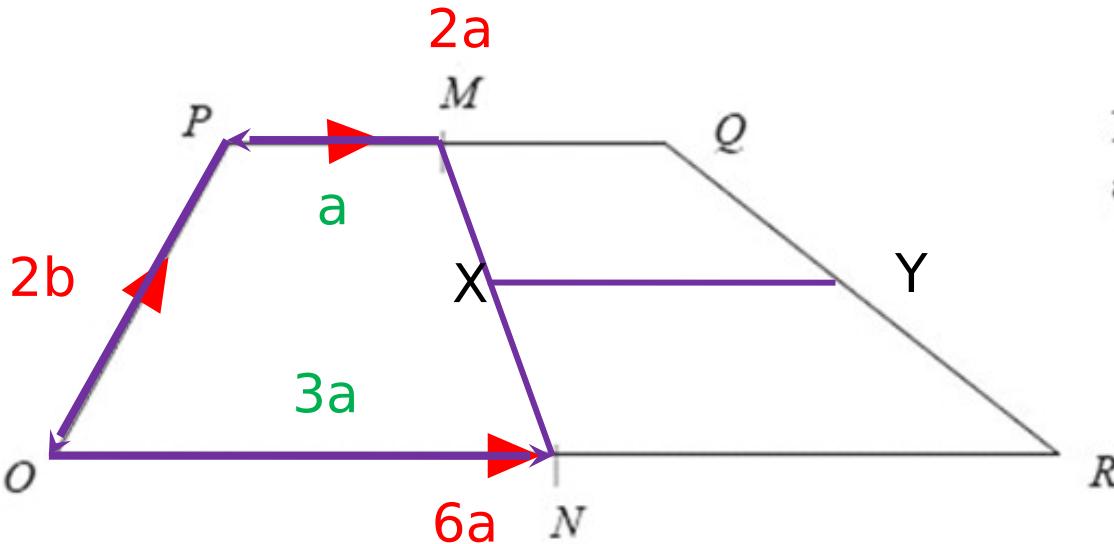


Diagram NOT  
accurately drawn

$OPQR$  is a trapezium with  $PQ$  parallel to  $OR$ .

$$\overrightarrow{OP} = 2\mathbf{b}$$

$$\overrightarrow{PQ} = 2\mathbf{a}$$

$$\overrightarrow{OR} = 6\mathbf{a}$$

$M$  is the midpoint of  $PQ$  and  $N$  is the midpoint of  $OR$ .

$$2\mathbf{a} - 2\mathbf{b}$$

(a) Find the vector  $\overrightarrow{MN}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$X$  is the midpoint of  $MN$  and  $Y$  is the midpoint of  $QR$ .

(b) Prove that  $XY$  is parallel to  $OR$ .

1<sup>st</sup> Work out QR

$$-2\mathbf{a} - 2\mathbf{b} + 6\mathbf{a}$$

$$4\mathbf{a} - 2\mathbf{b}$$

QY will be half this

$$2\mathbf{a} - \mathbf{b}$$

Find XY

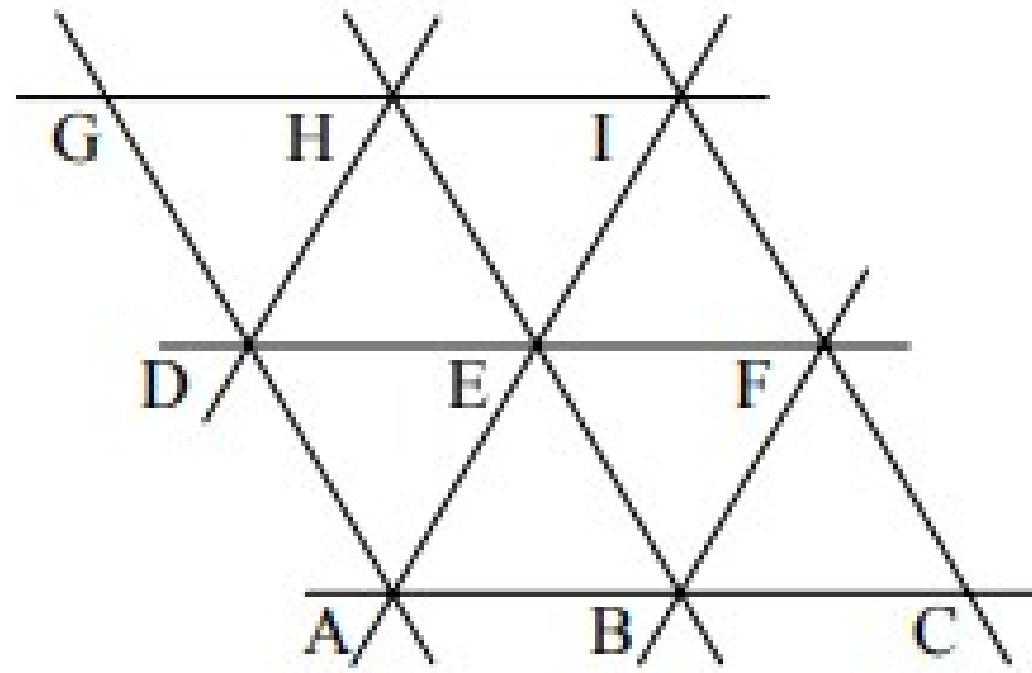
$$XM = \frac{1}{2}(-(2\mathbf{a} - 2\mathbf{b}))$$

$$XM = -\mathbf{a} + \mathbf{b}$$

$$XY = -\mathbf{a} + \mathbf{b} + \mathbf{a} + 2\mathbf{a} -$$

$$XY = \mathbf{b} 2\mathbf{a}$$

$\overrightarrow{AC} = \underline{s}$  and  $\overrightarrow{AD} = \underline{t}$



What could the question be?